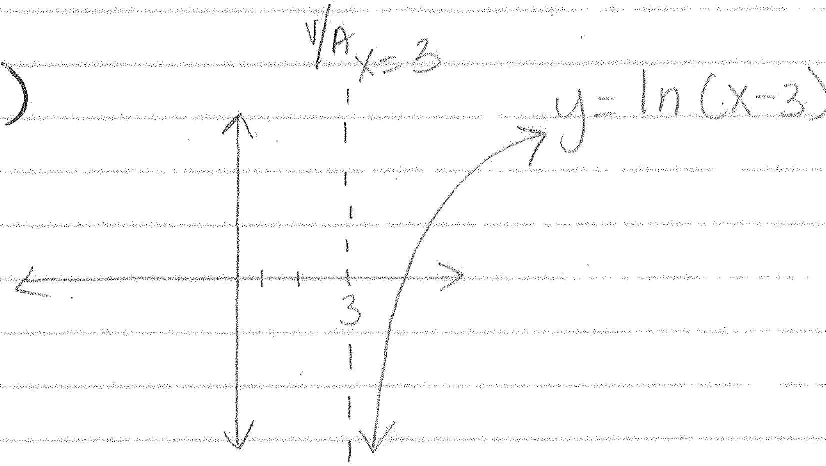
Friday, May 15th

5.1

#39 $\lim_{x \to 3^+} \ln(x-3)$

graph it!



$\lim_{x \to 3^+} \ln(x-3) = -\infty$, decreases without bound

#41 $\lim_{x \to 2^-} \ln[x^2(3-x)] = \lim_{x \to 2^-} [\ln x^2 + \ln(3-x)]$

$= \lim_{x \to 2^-} [2\ln x + \ln(3-x)]$

$= \lim_{x \to 2^-} 2\ln x + \lim_{x \to 2^-} \ln(3-x)$

$= 2\lim_{x \to 2^-} \ln x + \ln(3-2)$

$= 2\ln 2 + \ln 1 \leftarrow \ln 1 = 0$

$= 2\ln 2 + 0$

$= 2\ln 2$

#93 $y = x\ln x$, find any relative extrema and possible points of inflection.

$y' \longrightarrow$ critical numbers
$y'' \longrightarrow$ 2nd derivative test for rel. extrema and possible points of inflection

$$\frac{d}{dx}(y) = \frac{d}{dx}[x\ln x]$$

$$\frac{dy}{dx} = x\frac{d}{dx}(\ln x) + (\ln x)\frac{d}{dx}(x)$$

$$\frac{dy}{dx} = x \cdot \frac{1}{x} + \ln x \cdot 1$$

$$\frac{dy}{dx} = 1 + \ln x$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}[1 + \ln x]$$

$$\frac{d^2y}{dx^2} = \frac{1}{x}$$

Find Critical Numbers

(I) $\frac{dy}{dx} = 0$, or (II) $\frac{dy}{dx}$ is undefined

$1 + \ln x = 0$ never

$\ln x = -1$

$e^{\ln x} = e^{-1}$ ← gets rid of natural log (ln)

$x = \frac{1}{e}$

Friday, May 13th

5.2

#19 $\int \frac{x^4+x-4}{x^2+2}\,dx$

$$\begin{array}{r|l} & x^2 \quad -2 \\ \hline x^2+0x+2 & x^4+0x^3+0x^2+x-4 \\ & -x^4-0x^3-2x^2 \\ \hline & -2x^2+x-4 \\ & +2x^2 \quad +4 \\ \hline & x \end{array}$$

① $x^2(x^2)=x^4$
② $x^2(x^2+2)=x^4+2x^2$

① $x^2(-2)=-2x^2$

$=\int\left(x^2-2+\frac{x}{x^2+2}\right)dx$

$=\int x^2\,dx-2\int dx+\int\frac{x}{x^2+2}\,dx$

let $u=x^2+2$

$\frac{du}{dx}=2x$

$\frac{du}{2x}=dx$

$=\frac{x^3}{3}-2x+\frac{1}{2}\int\frac{1}{u}\,du$

$=\frac{x^3}{3}-2x+\frac{1}{2}\cdot\ln|u|+C$

$=\frac{x^3}{3}-2x+\frac{1}{2}\ln|x^2+2|+C$

using $(0, 2)$
$\uparrow$ $\uparrow$
θ S

$2 = -\frac{1}{2} \ln|\cos(2 \cdot 0)| + C$

$2 = -\frac{1}{2} \ln|1| + C$

$2 = -\frac{1}{2}(0) + C$

$2 = C$

#79 $y = 2\sec\left(\frac{\pi x}{6}\right) dx, \quad y=0 \quad x=0 \quad x=2$

$= \int_0^2 2\sec\left(\frac{\pi x}{6}\right) dx$

$= 2\int_0^2 \sec\left(\frac{\pi x}{6}\right) dx$ let $u = \frac{\pi x}{6}$

$= 2\int_0^2 \sec(u) \frac{6\,du}{\pi}$ $\frac{du}{dx} = \frac{\pi}{6}$

$dx = \frac{6\,du}{\pi}$

$= \frac{2(6)}{\pi} \int_0^2 \sec(u)\, du$

$= \left[\frac{12}{\pi} \ln\left|\sec\left(\frac{\pi x}{6}\right) + \tan\left(\frac{\pi x}{6}\right)\right|\right]_0^2$

$= \left[\left(\frac{12}{\pi} \ln\left|\sec\left(\frac{\pi}{3}\right) + \tan\left(\frac{\pi}{3}\right)\right) - \left(\frac{12}{\pi} \ln\left|\sec(0) + \tan(0)\right)\right]\right.$

$$f'(x) = \frac{\sin x \cos x}{1+\sin^2 x}$$

$$f'(x) = \frac{\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right)}{1+\sin^2\left(\frac{\pi}{4}\right)}$$

$$f'(x) = \frac{\frac{1}{2}}{1+\frac{1}{2}}$$

$$f'(x) = \frac{\frac{1}{2}}{\frac{3}{2}}$$

$$f'(x) = \frac{1}{3} = m_{tan}$$

$$y - y_1 = m_{tan}(x - x_1)$$

$$y - \ln\sqrt{\frac{3}{2}} = \frac{1}{3}x - \frac{\pi}{12}$$

$$y - \ln\sqrt{\frac{3}{2}} + \ln\sqrt{\frac{3}{2}} = \frac{1}{3}x - \frac{\pi}{12} + \ln\sqrt{\frac{3}{2}}$$

$$y = \frac{1}{3}x - \frac{\pi}{12} + \ln\sqrt{\frac{3}{2}}$$

Friday, May 13th

5.5

#51 $\log_4(5x+1)=y$, find $\frac{dy}{dx}$

$y=\frac{\ln(5x+1)}{\ln(4)}$ ⟵ change of base rule

$\frac{d}{dx}(y)=\frac{1}{\ln(4)}\cdot\frac{d}{dx}[\ln(5x+1)]$

$\frac{dy}{dx}=\frac{1}{\ln 4}\left(\frac{1}{5x+1}\right)\cdot\frac{d}{dx}(5x+1)$

$\frac{dy}{dx}=\frac{5}{\ln 4(5x+1)}$

#61 $g(t)=10\frac{\log_4 t}{t}\longrightarrow \log_4 t=\frac{\ln t}{\ln 4}$

$g'(t)=10\left[\frac{t\cdot\frac{d}{dt}\left(\frac{\ln t}{\ln 4}\right)-\left(\frac{\ln t}{\ln 4}\right)\frac{d}{dt}(t)}{t^2}\right]$

$g'(t)=10\left[\frac{\cancel{t}\cdot\left(\frac{1}{\ln 4}\right)\left(\frac{1}{\cancel{t}}\right)-\left(\frac{\ln t}{\ln 4}\right)(1)}{t^2}\right]$

$g'(t)=\frac{10}{\ln 4}\left[\frac{1-\ln t}{t^2}\right]$

$g'(t)=\frac{10(1-\ln t)}{\ln 4(t^2)}$

5.6

#45 $g(x) = 3\arccos\left(\frac{x}{2}\right)$

$$\frac{d}{dx}[\arccos(u)] = \frac{-u'}{\sqrt{1-u^2}}$$

$$g'(x) = 3\left[\frac{-1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\right]\frac{d}{dx}\left[\frac{x}{2}\right]$$

$$g'(x) = \frac{-3}{\sqrt{\frac{4}{4}-\frac{x^2}{4}}}\left(\frac{1}{2}\right)$$

$$g'(x) = \frac{-3}{\frac{1}{2}\sqrt{4-x^2}}\left(\frac{1}{2}\right)$$

$$g'(x) = \frac{-3}{\sqrt{4-x^2}}$$

OR

if you don't know

that $\frac{d}{dx}[\arccos(u)] = \frac{-u'}{\sqrt{1-u^2}}$

, then $\longrightarrow$

Ex. $\frac{d}{dx}[\text{arcsec}x]$

let $y = \text{arcsec}x$

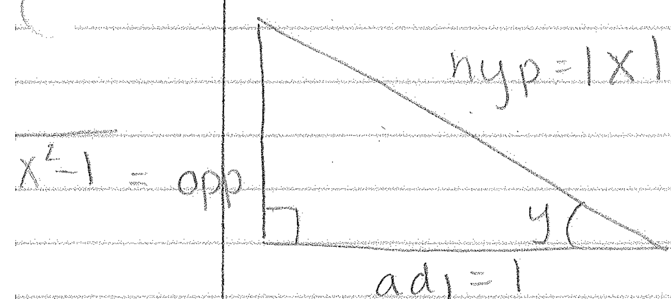
$\sec(y) = \sec(\text{arcsec}\,x)$

$\boxed{\sec(y) = x}$ implicit

$\frac{d}{dx}[\sec(y)] = \frac{d}{dx}(x)$

$\frac{dy}{dx}\sec(y)\tan(y) = 1$

$$\frac{dy}{dx} = \frac{1}{\sec(y)\tan(y)}$$



$\sec(y) = \frac{x}{1} = \frac{hyp}{adj}$

$\tan(y) = \frac{opp}{adj} = \frac{\sqrt{x^2-1}}{1}$

$|x| \cdot \sqrt{x^2-1} = \sec(y)\tan(y)$

$$\frac{1}{\sec(y)\tan(y)} = \frac{1}{|x|\sqrt{x^2-1}}$$

#101 $y = x\sqrt{x^2+1}$

$\ln(y) = \ln(x\sqrt{x^2+1})$

$\ln(y) = \ln x + \ln(x^2+1)^{1/2}$

$\ln(y) = \ln x + \frac{1}{2}\ln(x^2+1)$

$\frac{d}{dx}[\ln(y)] = \frac{d}{dx}\left[\ln x + \frac{1}{2}\ln(x^2+1)\right]$

$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{d}{dx}(\ln x) + \frac{1}{2}\frac{d}{dx}[\ln(x^2+1)]$

$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{x} + \frac{1}{2}\left[\frac{1}{x^2+1}\right]\frac{d}{dx}(x^2+1)$

$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{x} + \frac{1}{\cancel{2}}\left[\frac{1}{x^2+1}\right] \cdot (\cancel{2}x)$

$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{x} + \frac{x}{x^2+1}$

$y \cdot \frac{1}{y}\frac{dy}{dx} = \left[\frac{1}{x} + \frac{x}{x^2+1}\right] \cdot y$

$\frac{dy}{dx} = \left[\frac{1}{x} + \frac{x}{x^2+1}\right] x\sqrt{x^2+1}$

$\frac{d}{dx}\left[x\sqrt{x^2+1}\right] = \left[\frac{1}{x} + \frac{x}{x^2+1}\right] x\sqrt{x^2+1}$

2nd Derivative Test

Test $x = \frac{1}{e}$

$$\left.\frac{d^2y}{dx^2}\right|_{x=\frac{1}{e}} = \frac{1}{1/e}$$

$$= e$$

$$= e > 0 \longrightarrow$$

Relative minimum at $\left(\frac{1}{e}, -\frac{1}{e}\right)$

plug into original eq^{n}

Possible Points of Inflection

I $\frac{d^2y}{dx^2} = 0$ or, II $\frac{d^2y}{dx^2} =$ undefined

$\frac{1}{x} = 0$

$1 = 0$

False

$\frac{1}{x}$ is undefined at $x = 0$, but it is not in the domain.

— None —

#33 $\int \csc(2x)\, dx$

let $u = 2x$

$\frac{du}{dx} = 2$

$\frac{du}{2} = dx$

$= \int \csc(u) \left(\frac{du}{2}\right)$

$= \frac{1}{2} \int \csc(u)\, du$

$= \frac{1}{2}\left[-\ln|\csc(u) + \cot(u)|\right] + C$

$= -\frac{1}{2} \ln|\csc(2x) + \cot(2x)| + C$

#45 $\frac{ds}{d\theta} = \tan(2\theta)$ at $(0,2)$

$\int \frac{ds}{d\theta}(d\theta) = \int \tan(2\theta)\, d\theta$

$\int ds = \int \frac{\sin(2\theta)}{\cos(2\theta)}\, d\theta$

let $u = \cos(2\theta)$

$\frac{du}{d\theta} = -\sin(2\theta) \cdot \frac{d}{d\theta}(2\theta)$

$\frac{du}{d\theta} = -2\sin(2\theta)$

$\frac{du}{-2\sin(2\theta)} = d\theta$

$S = \int \frac{\sin(2\theta)}{u} \left(\frac{du}{-2\sin(2\theta)}\right)$

$S = -\frac{1}{2} \int \frac{1}{u}\, du$

$S = -\frac{1}{2}\left[\ln|u|\right] + C$

$S = -\frac{1}{2} \ln|\cos(2\theta)| + C$

$$= \left[\frac{12}{\pi}\ln(2+\sqrt{3})\right] - \left[\frac{12}{\pi}\ln(1+0)\right]$$

$$= \frac{12}{\pi}\ln(2+\sqrt{3}) - \frac{12}{\pi}\ln(1)$$

$$= \frac{12}{\pi}\ln(2+\sqrt{3}) - 12(0)$$

$$= \frac{12}{\pi}\ln(2+\sqrt{3})$$

5.1

\#79 $f(x) = \ln\sqrt{1+\sin^2 x} \quad \left(\frac{\pi}{4}, \ln\sqrt{\frac{3}{2}}\right)$

$$f(x) = \ln(1+\sin^2 x)^{1/2}$$

$$f(x) = \frac{1}{2}\ln(1+\sin^2 x)$$

$$f'(x) = \frac{d}{dx}\left[\frac{1}{2}\ln(1+\sin^2 x)\right]$$

$$f'(x) = \frac{1}{2}\left(\frac{1}{1+\sin^2 x}\right)\frac{d}{dx}(1+\sin^2 x)$$

$$f'(x) = \frac{1}{2}\left(\frac{1}{1+\sin^2 x}\right)(2\sin x)\frac{d}{dx}(\sin x)$$

$$f'(x) = \frac{1}{\cancel{2}(1+\sin^2 x)}(\cancel{2}\sin x)(\cos x)$$

Friday, May 13th

5.4

#125 $\int_0^{\pi/2} e^{\sin(\pi x)} \cdot \cos(\pi x)\, dx$

let $u = \sin(\pi x)$

$\frac{du}{dx} = \pi \cos(\pi x)$

$\frac{du}{\pi \cos(\pi x)} = dx$

$= \int_{u=0}^{u=\sin(\frac{\pi^2}{2})} e^{u} \cdot \cancel{\cos(\pi x)} \left(\frac{du}{\pi \cancel{\cos(\pi x)}}\right)$

$= \frac{1}{\pi} \int_0^{\sin(\frac{\pi^2}{2})} e^{u}\, du$

$x = 0$

$u = \sin(\pi \cdot 0)$

$u = \sin(0)$

$u = 0$

$x = \frac{\pi}{2}$

$u = \sin\left(\pi \cdot \frac{\pi}{2}\right)$

$u = \sin \frac{\pi^2}{2}$

$= \frac{1}{\pi} \left[e^{u}\right]_0^{\sin(\frac{\pi^2}{2})}$

$= \left[\frac{1}{\pi} \cdot e^{\sin(\frac{\pi^2}{2})}\right] - \left[\frac{1}{\pi} \cdot e^{0}\right]$

$= \frac{1}{\pi} \cdot e^{\sin(\frac{\pi^2}{2})} - \frac{1}{\pi}$

$= \frac{e^{\sin(\frac{\pi^2}{2})}}{\pi} - \frac{1}{\pi}$

$= \frac{e^{\sin(\frac{\pi^2}{2})} - 1}{\pi}$

#80 $\int (3-x)\, 7^{(3-x)^2}\, dx$

let $u = (3-x)^2$

$\frac{du}{dx} = 2(3-x)\frac{d}{dx}(3-x)$

$\frac{du}{dx} = 2(3-x)(-1)$

$\frac{du}{-2(3-x)} = dx$

$= \int \cancel{(3-x)}\, 7^u \left(\frac{du}{-2\cancel{(3-x)}}\right)$

$= -\frac{1}{2}\int 7^u\, du$

$= -\frac{1}{2}\left[\frac{7^u}{\ln 7}\right] + C$

$= -\frac{7^{(3-x)^2}}{2\ln(7)} + C$

#45 $\frac{d}{dx}[\arccos x]$

let $y = \arccos x$

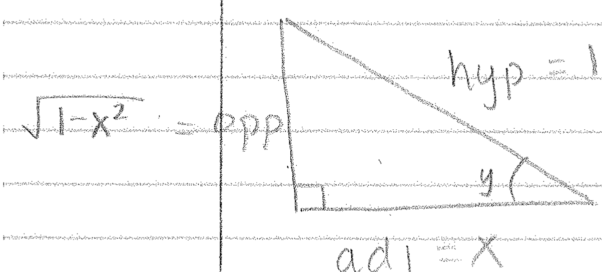
$\cos(y) = \cos(\arccos x)$

$\boxed{\cos(y) = x}$ implicit Differentiation

$$\frac{d}{dx}[\cos(y)] = \frac{d}{dx}(x)$$

$$-\sin(y)\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{-\sin(y)}$$



$$\cos(y) = \frac{adj}{hyp} = \frac{x}{1}$$

$$\sin(y) = \frac{opp}{hyp} = \frac{\sqrt{1-x^2}}{1}$$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$$